An Analysis of the AVE–SESAME I Period Using Statistical Structure
and Correlation Functions

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ABSTRACT

Structure and correlation functions are used to describe atmospheric variability during the 10–11 April
day of AVE–SESAME 1979 that coincided with the Red River Valley tornado outbreak. The special
mesoscale rawinsonde data are employed in calculations involving temperature, geopotential height, horizontal
wind speed and mixing ratio. Functional analyses are performed in both the lower and upper troposphere
for the composite 24 h experiment period and at individual 3 h observation times.

Results show that mesoscale features are prominent during the composite period. Fields of mixing ratio
and horizontal wind speed exhibit the greatest amounts of small-scale variance, whereas temperature and
geopotential height contain the least. Results for the nine individual times show that small-scale variance is
greatest during the convective outbreak. The functions also are used to estimate random errors in the
rawinsonde data. Finally, sensitivity analyses are presented to quantify confidence limits of the structure
functions.

1. Introduction

Compared to synoptic-scale systems, relatively little
is known about mesoscale meteorological features.
Phenomena such as moist convection, frontal rain-
bands, and boundary layer wind circulations are
important factors in determining local weather. In
fact, it is widely believed that major advances in
accuracy of precipitation forecasts will depend heavily
on the improved detection and understanding of
mesoscale systems (Committee on Atmospheric Sci-
ces, 1980). Inadequate data has been a major
limitation in mesoscale studies. The routine radio-
sonde network operated by the National Weather
Service (NWS) is too coarse for consistently describing
mesoscale phenomena because the stations have an
average spacing of 400 km and the observations are
taken at 12 h intervals.

The 1979 Atmospheric Variability Experiment–
Severe Environmental Storms and Mesoscale Exper-
iment (AVE–SESAME) was conducted in order to
gather additional mesoscale data, thereby permitting
continued investigations into subsynoptic-scale phe-
nomena (Lilly, 1975). During the first three experi-
ment days, a regional rawinsonde network over the
central United States provided information at 250
km spacings and 3 h intervals (meso α-scale); (Alber-
The 10–11 April day of AVE–SESAME '79 has
received considerable attention because it coincided
with the deadly Red River Valley tornado outbreak
(e.g., Alberthy et al., 1980; Anthes et al., 1982; Belt

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using the special soundings from AVE-SESAME '79. Structure and correlation functions are used to quantify magnitudes of gradients and random errors within the basic data in the lower and upper troposphere. Calculations are made for both the composite 24 h period and individual 3 h observation times. The following sections describe characteristics of the statistical functions, data and computational procedures, as well as the results.

2. Structure and correlation functions

Gandin (1963) used structure and correlation functions in statistical objective analysis procedures. Hillger and Vonder Haar (1979) present excellent discussions of these definitions. The structure value (STR) of a quantity $f$ at observation sites $r_1$ and $r_2$ is given by

$$\text{STR}(r_1, r_2) = \frac{1}{N} \sum \left( f'(r_1) - f'(r_2) \right)^2,$$

(1)

where $N$ is the number of observation times of the two sites,

$$f'(r) = f(r) - \bar{f}(r),$$

(2)

and $\bar{f}$ is the time mean for the $N$ observations at each location. A structure value is obtained for each unique pair of data points. Since each observation pair is considered only once and coincident points are not included, the total number of pairings for $L$ observation sites is $L(L - 1)/2$.

The assumption of homogeneity within a data field implies that values of structure depend only on the vector separation $r$ between points $r_1$ and $r_2$. Thus, structure is independent of the position of the station pair in the field, but dependent on their separation and orientation. When the further assumption of isotropy is made, the structure function becomes dependent only on the scalar separation distance $\rho$ between stations, i.e.,

$$\rho = |r_2 - r_1|.$$

(3)

Hence, the structure function for a station pair is independent of position and orientation in the field. Although the homogeneous and isotropic assumptions are frequently made in structure function analyses at the synoptic and mesoscales (e.g., Barnes and Lilly, 1975; Hillger and Vonder Haar, 1979), they are probably more applicable at the microscale.

The covariance function (COV) is defined as

$$\text{COV}(r_1, r_2) = \frac{1}{N} \sum \left\{ f'(r_1) f'(r_2) \right\}.$$

(4)

At zero separation, $r_1 = r_2 = r$, the covariance then equals the variance (VAR) at the point in question which is

$$\text{VAR}(0) = \frac{1}{N} \sum \left\{ f'(r) \right\}^2.$$

(5)

Homogeneous and isotropic fields generally are assumed, and the correlation function CORR then is obtained by normalizing the covariance function by the variance, i.e.,

$$\text{CORR}(\rho) = \frac{\text{COV}(\rho)}{\text{VAR}(0)}.$$

(6)

It should be emphasized that structure and correlation functions are random functions. The ergodic theorem states that, at least in principle, the information from a single synoptic situation is sufficient to determine the functional values. Gandin (1963) emphasized, however, that in practice one cannot use the ergodic theorem to study statistical properties of meteorological fields. He urged that many synoptic situations be included in the statistical analyses. Since the AVE-SESAME I period is a single severe storm event, the stationarity assumption must be invoked in the calculation of all mean quantities. Similar assumptions were needed in previous single event studies (e.g., Maddox and Vonder Haar, 1979; Maddox, 1980).

The formulations of structure and correlation functions described here require time means at the fixed observing sites. However, this is usually not possible with satellite data because different locations are sampled with each pass. The procedure also is not feasible with rawinsonde data if only one observation time is available. To circumvent these limitations, Hillger and Vonder Haar (1979) and Maddox and Vonder Haar (1979) have utilized an alternate procedure such that horizontal means at single times are employed. On the assumption of homogeneous, isotropic conditions, the new definition of the structure function is

$$\text{STR}(\rho_i) = \frac{1}{K} \sum \left\{ f(r_1) - f(r_2) \right\}^2,$$

(7)

where $K$ is the number of station pairs within separation intervals ($\rho_i$) such as 150 km (i.e., 0–150, 151–300 km, etc.). Thus, the new procedure is to compute one structure value for all data pairs within each separation interval.

Notice that $f'$ can be used instead of $f'$ in (7) since the horizontal means are assumed to cancel. However, this hypothesis may not be entirely appropriate for particular synoptic situations (Barnes, personal communication, 1983). Although deviations from time mean values for different stations (original procedure) tend to have similar structure for long time periods, deviations from space means for only one synoptic situation (modified procedure) do not average to similar values at different stations. Therefore, in (7), the mean value that is implicitly subtracted is not necessarily the appropriate mean for the given station pair during the time interval under consideration. This may contribute to some of the structure that is calculated from this procedure.
The new definition of the correlation function for homogeneous, isotropic conditions is

$$\text{CORR}(\rho_j) = \frac{\frac{1}{K} \sum [f'(r_1) f'(r_2)]}{\frac{1}{M} \sum f'(r_1)^2}.$$  (8)

Values of $f'$ are deviations from the horizontal mean ($\bar{f}$) within each separation interval. In computing the variances (the denominator), we used stations within each particular interval only once, regardless of the number of times that they were utilized in obtaining the covariance (numerator). Thus, $M$, the number of stations used for the variance, usually differed from $K$, the number of station pairs. Alternative normalization schemes are described by Barnes and Lilly (1975) and Maddox and Vonder Haar (1979).

Structure and correlation functions possess several useful characteristics. Hillger and Vonder Haar (1979) noted that the slope of the structure profile is a measure of mean nondirectional gradients within a data field. That is, rapid changes in structure with increasing station separation are indicative of large horizontal gradients. Without the assumption of isotropy, structure functions indicate mean orientations of gradients.

An additional attribute of structure functions is their measure of random error within a homogeneous, isotropic data set (Hillger and Vonder Haar, 1979). Errors are estimated by extrapolating structure curves to zero separation distance, i.e.,

$$\text{STR}(0) = 2\sigma^2,$$  (9)

where $\sigma^2$ is the mean-squared uncertainty. Structure-derived values do not include bias or systematic errors, and a major advantage of the procedure is that results are not based on comparisons with other types of data. For example, to obtain random error estimates of satellite data, it is not necessary to compare them with their rawinsonde-derived counterparts.

Correlation functions provide information somewhat similar to that of structure functions. However, since correlation values are normalized to ±1, this parameter readily permits results from one type of data to be compared with those from others. Because of random data errors, correlation values are not intrinsically 1.0 at zero separation (Hillger and Vonder Haar, 1979). This occurs because the computed variance at zero separation, $\text{VAR}^*(0)$, contains the effects of the uncertainty, i.e.,

$$\text{VAR}^*(0) = \text{VAR}(0) + \sigma^2.$$  (10)

The resulting correlation function that accounts for random errors is

$$\text{CORR}^*(\rho) = \frac{\text{CORR}(\rho)}{1 + [\sigma^2/\text{VAR}(0)]}.$$  (11)

If data uncertainty ($\sigma^2$) is estimated from the structure function [as in (9)], (11) can be used to correct the correlation results such that the values are 1.0 at zero separation. In this way CORR describes meteorological variability only, and not an additional factor that is due to data noise. Correlation functions of the current study were adjusted using the previously mentioned procedure.

3. Data and synoptic conditions

The first regional-scale day of AVE–SESAME '79 extended from 1200 GMT 10 April through 1200 GMT 11 April and included rawinsonde releases at 3 h intervals from 23 NWS stations plus an additional 16 special sites (Fig. 1). The average station spacing was 250 km. Data were checked carefully for errors prior to use. Special attention was given to soundings thought to have questionable geopotential heights or arising from sondes having variable ascent rates, possibly due to storm penetration (Gerhard et al., 1979; Barnes, 1981). Clearly erroneous data were not used in subsequent calculations. The data that were incorporated appear to be representative of the meso-$\alpha$-scale storm environment.

Data handling procedures were similar to those of Fuehrberg and Jedlovec (1982); therefore, only highlights will be given here. To correct for nonsimultaneous sonde releases and variable ascent rates, data at all levels were adjusted to a common time using a linear interpolation scheme similar to that of Fankhauser (1969). Since most sondes were released 55 min prior to the standard hour (e.g., 1105 GMT for 1200 GMT), all data were adjusted to the prior hour, although they will be denoted as 1200, 1500, etc., (GMT), soundings. When soundings were missing,
data were generated by time interpolation from the available releases if the gap was no greater than 6 h. Extrapolation was not utilized; thus, soundings were not generated at the first or last observation times. Our goal was to use as large a data set as possible; however, with the abovementioned restrictions, only about 7% of the total soundings used were mathematically generated. This fact, coupled with the sensitivity analyses of Section 4e suggest that the procedure may not have been necessary. It has the effect of slightly smoothing the statistical results that follow. To verify the final data set, time series and constant pressure analyses were performed. The data are identical to those used by Fuelberg and Jedlovec (1982).

Detailed analyses of weather conditions during the AVE-SESAME I period have been given by Alberty et al. (1980), Carlson et al. (1980), Moore and Fuelberg (1981), and Vincent and Homan (1983). Figure 2 shows radar echoes and map features near 0000 GMT 11 April, approximately midway through the 24 h experiment period. The surface pattern (Fig. 2a) is dominated by a low over Colorado. A cold front extends southward, while a warm front stretches along the Red River Valley and into the Gulf of Mexico. A dry line is located over western Texas. At 700 mb (Fig. 2b), winds are predominantly from the southwest. A low-level jet stream had formed over central Texas near 2100 GMT. Although it is centered near 850 mb, its effects are quite apparent at 700 mb where, at 0000 GMT, winds greater than 30 m s⁻¹ are located over the Red River Valley. At 300 mb (Fig. 2c), jet intrusion from the southwest is evident over West Texas; winds over El Paso (ELP) are approximately 50 m s⁻¹. An area of relatively light winds is found over northcentral Texas, while a secondary maximum exists over Oklahoma and Kansas. Time variations in the upper-level flow and its relation to the convection are described by Moore and Fuelberg (1981) and Fuelberg and Jedlovec (1982).

The most intense convective activity occurred between 2100 GMT 10 April and 0600 GMT 11 April over Texas and Oklahoma. At 2335 GMT (Fig. 2a) echo tops reached 17.7 km (58 000 ft) near Wichita Falls (SPS) where the devastating tornado touched down at 0000 GMT 11 April. Less intense convection covers much of the remainder of the experiment region.

4. Results

a. Composite period

The initial task was to compute structure and correlation functions via the approach given in (1)-(6). Calculations were made for 850, 700, 300 and 200 mb in order to describe both lower and upper tropospheric features. Since average sonde drift at 100 mb was 90 km, compared to the typical station spacing of 250 km, all computations utilized sonde positions at particular levels instead of the surface station locations. The formulation utilizes time means at the individual sites, and the procedure for computing these values is critical. If data were missing for either station of the pair at a particular time, data at the other station (if available) also were not used in the computation of means. Furthermore, if data were not available for fewer than eight of the nine observation times, no calculations were made. Since 39 rawinsonde sites participated in the experiment, a maximum of 741 values could be computed at each level. Missing data were relatively infrequent; there-

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**Fig. 2.** Radar summary for 2335 GMT 10 April 1979 (echo tops in kilometers). Surface fronts (National Meteorological Center) and upper-level maps (from AVE-SESAME data) are for 0000 GMT 11 April. The surface position of the dry line is dashed while dashed lines at 700 and 300 mb are isotachs (2 = 20 m s⁻¹). At 700 mb, 06 means 3060 m, while at 300 mb, 06 means 9060 m.
fore, the actual number of station pairs was only slightly less than the maximum.

Structure and correlation functions of geopotential height at 700 mb are shown in Fig. 3. Values are plotted with respect to scalar station separation, signifying the assumptions of homogeneity and isotropy. One expects structure values to increase with greater station (sonde) separation. On the other hand, values of correlation should be near 1.0 at the smallest separations and then decrease as the sites become more distant. In general, the calculated results show the expected distribution. Although some point values exhibit rather extreme scatter and may initially appear conspicuous, they have been verified by extensive hand calculations.

It is informative to consider the circumstances producing particular values of structure and correlation. Point A of Fig. 3 represents the pairing of Peoria, Illinois (PIA) and Columbia, Missouri (COU) which are only 293 km apart (Fig. 1). Since the structure value (Fig. 3) appears somewhat large (298 m²), while the correlation value is rather small (~0.50), substantial mesoscale activity is indicated. A time series of 700 mb heights for these stations (Fig. 4) demonstrates this variability. Profiles are quite similar through the fifth observation (0000 GMT 11 April);

![Diagram](image)

**Fig. 4.** Time series of geopotential height at 700 mb for station pairs A, B, and C of Fig. 3. Dashed lines show time means of the particular stations. Data are at 3 h intervals.

Fig. 3. Structure function (top) and correlation function (bottom) for geopotential height at 700 mb.

however, they differ considerably for the final four times. Deviations from each station's time mean tend to be of opposite sign which, according to (1) and (4), produce a relatively large value of structure and a negative covariance and correlation. Vincent and Homan (1983) have documented large subsynoptic-scale surface pressure tendencies in central Missouri after 0000 GMT 11 April in association with the thunderstorm activity. Thus, differences in the time series of 700 mb heights may be indicative of small scale height variations within the convective environment.
The situation at Point B (Fig. 3) is similar to that of Point A. Point B represents the pairing of Stephenville, Texas (SEP) and Nashville, Tennessee (BNA) which are 1155 km apart. Time series for these stations (Fig. 4) indicate variations that are considerably larger and more out of phase than those of Point A. As a result of the differences between the two stations, the calculations produced a large value of structure (1250 m$^2$), together with a negative correlation ($-0.85$). Two surface mesolows passed near Stephenville after 1800 GMT 10 April (Moore and Fuelberg, 1981; Vincent and Homan, 1983), and thunderstorm activity in the area was quite intense. On the other hand, Nashville did not experience the intense convection or pronounced surface mesoscale pressure fluctuations.

Point C (Fig. 3) differs considerably from Points A and B. In this case, small structure (103 m$^2$) and large correlation (0.82) occur even though the stations of Boothville, Louisiana (BVE) and Denver, Colorado (DEN) are 1823 km apart. These values result from a similarity in the time series of the two stations (Fig. 4). Both locations were outside the region of major convective activity and large surface pressure variations (Vincent and Homan, 1983).

Structure and correlation functions of height at 850, 300 and 200 mb (not shown) generally are similar to those at 700 mb. In addition, functions of temperature (not shown) agree closely with those of height since the height values were hydrostatically obtained from thermal data (Gerhard et al., 1979). On the other hand, structure and correlation functions of wind speed and humidity exhibit even more scatter than observed earlier. As an example, Fig. 5 shows functions of scalar wind speed at 700 mb. When compared with those of height (Fig. 3), one should note the greater frequency of points having large structure and small or negative correlation at small station separations. As noted earlier, such values indicate pronounced mesoscale activity such as the upward extension of the low-level jet stream (Fig. 2) or convectively influenced local winds. An additional consideration may be a less favorable signal to noise ratio for the wind data.

In an attempt to investigate the scatter of the structure and correlation functions, additional procedures were tested. Although the previously described results were based on removal of the time mean from each station, time trends due to the larger scale system over the 24 h period were included. Since trends at distant stations might differ considerably, it was hypothesized that this factor might contribute to some of the scatter. Therefore, structure and correlation functions were recomputed, this time with the station time mean and the linear trend removed independently at each site. Although Barnes (personal communication, 1983) found that this procedure yielded smoother functions when using NSSL surface observations, our results based on the AVE–SESAME RAOB data (not shown) were not significantly improved. Additional factors that may produce some of the observed scatter are the single synoptic case and the assumptions of homogeneity and isotropy. A later section will demonstrate that atmospheric patterns during this period were strongly anisotropic.

Although the formulation described above produces functional values for individual station pairs, the amount of scatter makes it difficult to quantify overall atmospheric structure. One solution is to average values obtained from (1)–(6) over intervals of station separation, obtaining a single curve. This procedure could be useful for investigating the entire 24 h period; however, it does not permit computations at individual observation times which was one of our goals (Section 4d). Therefore, we used the modified procedure involving horizontal means [(7)–(8)] for both the combined 24 h and individual 3 h data sets. The technique requires the selection of a separation interval. Although intervals ranging from 90 through 210 km were investigated, a value of 150 km was chosen because it produced results which appeared neither too "noisy" nor too smooth.
Fig. 6. Structure functions of geopotential height for the composite 24 h period. Values at the top represent numbers of station pairings within each separation interval at 700 mb.

Fig. 7. As in Fig. 6 except for temperature.

Fig. 8. As in Fig. 6 except for wind speed.

Figures 6–10 present structure functions of height, temperature, wind speed, and mixing ratio for the composite 24 h period, i.e., the combination of the nine individual 3 h observation times. For each of the four levels on the diagrams, a structure value is plotted midway between each separation interval of 150 km. The most distant separation category depicted is 1650–1800 km because few station pairs are located farther apart. Values at the top of each figure represent the number of pairings within each interval at 700 mb. Pairings at 850 mb are approximately 10% smaller because this level is below ground at some of the western stations. Similarly, pairings at 300 and 200 mb are decreased approximately 10 to 15%, respectively, from those at 700 mb due to early termination of some soundings. A later section will describe the sensitivity of structure results to the number of available stations. Briefly stated, however, the number of pairings at all but the first and last categories appears sufficient for the types of interpretations that follow.

Figure 6 shows that structure profiles of height in the upper troposphere have a much greater slope than those nearer the surface. Thus, mean nondirectional height gradients are stronger in the upper levels. This fact is verified by average wind speeds over the network which increase from 16.3 m s\(^{-1}\) at 850 mb to 40.7 m s\(^{-1}\) at 200 mb. At each particular level, the slopes are fairly smooth, suggesting that there are no specific wavelengths having pronounced variance.

Structure functions of temperature (Fig. 7) indicate substantial differences in gradients between the four levels. Strongest gradients (steepest slopes) are found at 850 mb and are probably attributable to frontal activity over the area (Fig. 2) as well as terrain influences. Horizontal gradients at 200 mb are stronger than those at 300 mb, and undulations in the 200 mb structure also are more pronounced than at any other level. The undulations imply greater horizontal temperature variability at some wavelengths than others. Constant pressure maps (not shown) confirm the stronger overall gradients at 200 mb and reveal numerous small scale cold and warm tongues which are probably due to either variations in tropopause height or convective influences.

Horizontal gradients of scalar wind speed are strongest at 200 mb (Fig. 8). At both 300 and 200 mb, structure peaks near separations of 500 and 800 km document mesoscale wind features that have been described by Moore and Fuelberg (1981) and Fuelberg and Jedlovic (1982). Mean nondirectional gradients
at 850 and 700 mb are similar to each other but are much smaller than those in the upper troposphere. An enlarged structure diagram for 700 mb is shown in Fig. 9. One should note the steep slope at separations below approximately 500 km. This indicates large horizontal gradients in mesoscale wind patterns corresponding mostly to the upward extension of the low-level jet stream (Fig. 2). A later section will describe temporal changes in structure that accompany the development of the jet.

Finally, structure functions for mixing ratio are shown in Fig. 10 for 850 and 700 mb only, since humidity data were unavailable above 350 mb. Structure values for the first separation interval (0–150 km) are plotted but not used to define the profile since the values appear unacceptably large. Apparently, the nine station pairings for this interval (one pair for nine observation times) are insufficient for a parameter with such highly variable horizontal gradients. The important feature of Fig. 10 is the steep slope (large gradient) at 850 mb that occurs at both mesoscale and synoptic scale. This corresponds to a dry line and frontal activity over the region as well as numerous small-scale areas of moist and dry air (Moore and Fuelberg, 1981). The small structure near 1800 km may be due to a small number of stations oriented in a particular direction. Gradients at 700 mb are considerably weaker than those at 850 mb.

To investigate the isotropic assumption incorporated in the previous results, structure functions were recomputed using horizontal averages as before. However, this time results also were categorized according to directions of separation vectors between the station pairs. Intervals of 30° were used, similar to that of Maddox and Vonder Haar (1979). Figure 11 shows that 700 mb heights are highly anisotropic during the composite 24 h period. Greatest variance occurs along a northwest–southeast direction, approximately perpendicular to the area-averaged flow (Fig. 2b). Similar features are observed for wind speed at 700 mb (Fig. 12), but considerably more mesoscale detail is apparent. In the case of mixing ratio (Fig. 13), the greatest variance occurs along a northeast–southwest axis. This orientation is normal to the isopleths of vapor content associated with the combination of the dry line and the warm front (Fig. 2b). Plots for levels and parameters not shown also reveal strongly anisotropic conditions; however, diagrams indicate that the isotropic assumption is best satisfied at the smallest separation categories.

b. Uncertainty estimates

Structure functions for the composite 24 h period (Figs. 6–10) were extrapolated to zero separation distance, in order to obtain random error estimates for each parameter [see (9)]. Table 1 presents these results as a function of pressure along with corresponding means and standard deviations of the data from which they were derived. The procedure was to use least-squares techniques to fit the following function to the structure values

\[
\text{STR}(x) = B \exp(ax),
\]

where \(x\) is the separation distance, \(a\) the coefficient of curvature, and \(B\) the intercept value at \(x = 0\). For all parameters except mixing ratio, two fits were made. The first was based on structure values from the first four separation categories (through the 450–600 km interval), while the second was derived from
Table 1. Root-mean-square errors with means and standard deviations (std dev) for the composite 24 h period. Errors in parentheses are from Hoehne (1980), those in brackets are from Lenhard (1970) and those in braces from Kurihara (1961).

<table>
<thead>
<tr>
<th>Level (mb)</th>
<th>Temperature (°C)</th>
<th>Height (m)</th>
<th>Wind speed (m s⁻¹)</th>
<th>Mixing ratio (g kg⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std dev</td>
<td>rms error</td>
<td>Mean</td>
</tr>
<tr>
<td>200</td>
<td>-57</td>
<td>4.1</td>
<td>1.3</td>
<td>(0.6)</td>
</tr>
<tr>
<td>300</td>
<td>-44</td>
<td>3.1</td>
<td>0.7</td>
<td>(0.6)</td>
</tr>
<tr>
<td>700</td>
<td>3</td>
<td>4.5</td>
<td>1.0</td>
<td>(0.6)</td>
</tr>
<tr>
<td>850</td>
<td>11</td>
<td>5.0</td>
<td>2.0</td>
<td>(0.6)</td>
</tr>
</tbody>
</table>

The first five categories. The two values of 2σ² obtained at zero separation then were averaged to produce the estimates of σ in Table 1. In the case of mixing ratio, structure values at the first separation category were not used since they appeared much too large (Fig. 10). As a result, error values were obtained from a single exponential fit for categories 2 through 5. Although linear functions have been used to estimate errors in previous studies involving meso β-scale data (Hillger and Vonder Haar, 1979; Maddox and Vonder Haar, 1979), the exponential function (12) produced the best fits during AVE–SESAME I.

Random error estimates of temperature (Table 1) are approximately 1°C. These values are somewhat greater than those obtained from paired, simultaneous radiosonde flights (Hoehne, 1980; Lenhard, 1970). Uncertainties in height range from 13 m at 850 mb to 28 m at 200 mb. They generally agree with those of previous studies and exhibit the expected increase with altitude attributable to integration of virtual temperature over progressively deeper layers.

Estimates of random wind errors generally increase with altitude (Table 1), ranging from 2.8 m s⁻¹ at 700 mb to 6.2 m s⁻¹ at 200 mb. Kurihara (1961) has attributed this trend to various sonde tracking limitations that become more pronounced at low sonde elevations occurring at high altitudes and large horizontal displacements from the release sites. Current error estimates are somewhat greater than those of Kurihara (1961), which are similar to findings of Danielsen (1959) and Fuelberg (1974). However, our greater values should be expected since jet level winds during AVE–SESAME I were quite strong. Sonde elevation angles in the upper levels were frequently less than 10° (Gerhard et al., 1979), considerably below optimum values that would yield smaller uncertainty. Finally, error estimates of mixing ratio are approximately 1.75 g kg⁻¹.

The random error values of Table 1 should be interpreted as estimates. Since the current study deals only with a single case, additional data sets may yield somewhat different statistical values. Although the isotropic and homogeneous assumptions were employed as in previous studies of this kind (Hillger and Vonder Haar, 1979; Maddox and Vonder Haar, 1979), Figs. 11–13 demonstrated that fields were highly anisotropic, although they did approach isotropy at the smaller separation categories utilized in the error calculations. A final consideration is our assumption of no spatially correlated errors. Maddox and Vonder Haar (1979) indicated that correlated spatial errors would decrease the magnitude of the estimated random errors.

![Figure 11](image_url)

**Fig. 11.** Anisotropic structure function of geopotential height at 700 mb for the composite 24 h period. Structure units are 10⁷ m², while separations are 10⁵ km.
Error estimates at 850 mb (Table 1) appear somewhat large when compared to those at 700 mb, and this may be due to at least two factors. First, there were approximately 10% fewer station pairs at 850 mb than at 700 mb because of the higher terrain in the western part of the network. Thus, the greater error estimates may be due, in part at least, to the relatively smaller amount of data. A second factor is that a portion of the low-level atmospheric variability may have been mistakenly interpreted as random error. This might occur due to the data being too coarsely spaced or to limitations in the error estimation techniques. Of course, the effects of this limitation may appear at all levels, not just the lowest.

Although one should consider the cautionary notes, the important point is that the procedure appears to be a useful method for obtaining error estimates that has not been utilized to maximum advantage.

c. Comparison of parameters

It is informative to compare the atmospheric variability contained in the four parameters being investigated. Correlation functions are ideally suited for this purpose since they are normalized to ±1, thereby eliminating differences in absolute magnitude that appear in structure functions.

Correlation values based on (8) contain the effects of random errors as well as true atmospheric variations. Based on the work of Hilliger and Vonder Haar (1979), the effects of random errors were removed through the use of (11). Values of \( \sigma \) needed in (11) were taken from Table 1. Because of the scatter with individual correlation values, the next step was to fit mathematical functions to the adjusted correlation data using least-squares procedures. The function employed was

\[
\text{CORR}(x) = \exp(-ax^2).
\]  

(13)

It possesses a desirable property of a correlation function, i.e., producing a positive definite spectrum (Gandin, 1963). Other investigators utilizing similar functions include Buell (1972), Lipton and Hillger (1982), and Schlatter (1975). Adjusted correlation values and results of the curve fitting procedure are shown in Fig. 14 for the four pressure levels of interest. One should note that the exponential functions are accurate representations of the data in most cases. Although correlation values at the smallest separation category (0–150 km) do not approach 1.0 as desired, sensitivity analyses indicate that values in the first category have only minimal effects on the resulting curve fits because the exponential function (13) intrinsically produces unity at zero separation. The apparent cause for the unexpected values in the first category is the limited amount of data that is available. Figures 6–10 indicate that only nine station pairings were involved in calculations for the first interval, although 400 to 800 pairings typically were used in the remaining categories. Additional sensitivity analyses revealed that curve fits were affected little by minor variations in the random errors utilized in (13).

Mixing ratio clearly is the most variable parameter (Fig. 14), since values of its correlation function are smallest. For example, at 400 km, the typical spacing of the NWS rawinsonde network, values are only 0.2–0.4. Correlations then decrease to zero separation at only 700–800 km. Thus, in the mean only adjacent values within this distance would be positively correlated. On the other hand, the parameter with the least amount of variability (greatest correlations) is geopotential height. In this case correlations at 400
km are approximately 0.7, and positive values still occur at separations well beyond 1000 km. As one might expect, temperature patterns exhibit smaller correlations than do those of height. Finally, horizontal winds are highly variable, since correlations are small and approach zero at ~1000 km.

The results confirm that a great deal of the atmospheric structure occurs at subsynoptic scales during the convectively active AVE–SESAME I period. This is especially true of mixing ratio and wind speed where correlations decrease rapidly at small separation distances. Barnes and Lilly (1975) obtained similar findings from NSSL data. If the special AVE–SESAME rawinsonde sites had not been available, much of the variance below 400 km separation would not have been resolved. Even with the special network, small scale features below 250 km separation are not detected. Thus, we reach the widely stated conclusion that mesoscale data are especially valuable during severe storm outbreaks.

d. Structure at individual times

Three hourly constant pressure maps of the AVE–SESAME I period reveal pronounced short term atmospheric changes (Moore and Fuelberg, 1981; Vincent and Homan, 1983). It is informative to examine these developments using structure function analyses. Our methodology was similar to that employed for the composite computations, i.e., (7) was used. With the current approach, however, each observation time was calculated separately. It should be noted that the assumptions of stationarity, homogeneity, and isotropy are less valid for single times than for the composite period. In addition, a smaller amount of data (fewer station pairings) now is available. Nonetheless, the sensitivity analyses of Section 4e indicate that there is sufficient data for the interpretations that follow.

Figure 15 depicts structure functions of 700 mb height at individual observation times. Only profiles at 6 h intervals are given for clarity of presentation. The major feature is that structure functions achieve greater slopes as the period progresses, signifying stronger mean nondirectional gradients. However, except for the changing slopes, the various curves are remarkably similar.

To further investigate the height fields, structure functions of 3 h height changes (Δz) were prepared. Figure 16 shows that mesoscale gradients of height tendency at 700 mb are weak at most times. However, the profile for 2100–1800 GMT 10 April exhibits a steep slope, indicating a strongly enhanced gradient. Peak values, occurring at a separation of approximately 600 km, correspond to a region of strong height falls over the Red River Valley of Texas and Oklahoma that preceded the tornado outbreak (Fig. 17). Greatest falls of 58 m occurred at Childress,
Texas (CDS). Although localized height changes over this area were quite large, the resulting trough was not pronounced and covered only a small portion of the analysis region (Fig. 2b). Thus, variations in mean nondirectional height gradients are slight (Fig. 15).

Structure functions of wind speed at 700 mb exhibit impressive fluctuations (Fig. 18). The most interesting point is the development of strong speed gradients that are largest at 0300 GMT 11 April. Horizontal maps (Fig. 2c) clearly reveal the mesoscale feature denoted by the functions. Between 2100–0300 GMT a low-level jet stream developed in the southwesterly flow between the Texas panhandle and Arkansas. Although centered at 850 mb, the 700 mb winds at Oklahoma City (OKC) increased from 16 to 35 m s⁻¹ during a 3 h period. Strongest values at 0300 GMT were 38 m s⁻¹ at Fort Smith, Arkansas (FSM). It is noteworthy that the 2100–0300 GMT period coincided with much of the tornado activity of the outbreak.

Well-defined areas of velocity divergence accompanied the Red River Valley tornado outbreak (Moore and Fuelberg, 1981), and structure functions are an ideal method for quantifying these patterns. The procedure was first to analyze objectively the data onto a grid encompassing the experiment region. The Barnes (1964) technique was employed on a grid mesh of 127 km. Divergence then was calculated using centered finite differences. Complete details about these procedures are given in Fuelberg and Jedlovec (1982). Structure functions were obtained from (7) at individual observation times. Grid point values were used instead of station data. A separation interval of 100 km was employed, and the number of data pairings often was in excess of 1000.

Figure 19 presents structure functions of velocity divergence at 700 mb. At 1500 GMT, the nearly horizontal profile indicates that mean nondirectional gradients are weak. By 2100 GMT, however, the start of convective outbreak, gradients of divergence have increased substantially. Strong gradients continue through the remainder of the period. One should note that considerable variance occurs at mesoscale separations. This supports results of Belt and Fuelberg (1982) who calculated fields of divergence, vorticity advection, and other kinematic parameters during the 10–11 April period. Although their patterns, derived from NWS data alone, had the same largescale features as those obtained from the higher
resolution data, the NWS network did not reveal the numerous small scale details. One should recall that correlation functions of wind speed (Fig. 14) had indicated significant variance at the mesoscale. Thus, the pronounced small scale variability in divergence (Fig. 19) is to be expected. Horizontal variations in both wind speed and direction lead to divergence, and the effects of wind direction have not been explored in the current analysis. Thus, temporal changes in the structure of divergence (Fig. 19) do not always correspond exactly with those of wind speed (Fig. 18).

\textit{e. Sensitivity analyses}

Sensitivity analyses were conducted to evaluate the effects of missing data and data errors on the structure results. The experiments were performed at a single observation time instead of the composite period because all effects would be more pronounced with the smaller data set. The time 0000 GMT 11 April was selected because it had well-defined structure profiles and because few stations were missing.

The first experiment consisted in removing the data of three stations from the input file. Eight different sets of station trios were obtained through random selection. Two additional stations [Bartlesville (BVO) and Durant (DUA)] were missing originally at 0000 GMT; thus, five of the 39 possible input values were unavailable for calculations. This was the maximum amount of missing data at any of the observation times.

Results for wind speed at 700 mb are shown in Fig. 20, where the original pattern with two missing sites (also shown in Fig. 18) is denoted by the heavy solid line. The important point is that structure slopes and locations of structure maxima are relatively unaffected by the missing data. Structure values at individual separations show somewhat greater effects; however, that information was not used extensively during this investigation. Results for other parameters and other levels are similar to those of Fig. 20. Thus, we are confident that our interpretations concerning the individual observation times are not compromised by the amount of missing data therein. In other
words, 34 stations are sufficient for diagnosing major changes in mean nondirectional atmospheric gradients at single observation times. Since calculations for the composite period are based on approximately nine times as much information as those at single times, it is even less likely that features of those functions are due to insufficient data.

The second experiment was designed to evaluate the effects of input data errors on structure results. The procedure was similar to that employed in sensitivity analyses of energy budget parameters (e.g., Vincent and Chang, 1975; Fuelberg and Jedlovec, 1982; Lin and Smith, 1982). That is, random perturbations simulating rawinsonde errors were added to original sounding data (which also contained error). Then, derived structure results were compared with those from the unaltered data. The 0000 GMT 11 April observation again was selected, and eight perturbed versions of data were prepared. The resultant perturbed data set is identical to that used by Fuelberg and Jedlovec (1982).

As an example of these calculations, results for wind speed at 700 mb are shown in Fig. 21. Based on Kurihara (1961) and Fuelberg (1974), standard deviations for the deliberately introduced, normally distributed errors are assigned to be 4.0° for wind direction and 2.0 m s⁻¹ for wind speed. One should note that structure functions derived from the altered data have the same major features as those from the original data set (heavy solid line). Since the procedure is to add random errors to data that already contained them, Fig. 21 shows that structure values at the smallest separation are greater for the perturbed versions than for the original. Thus, if the profiles were extrapolated to zero separation distance as done in Section 4b, the perturbed versions would show the expected greater errors. The conclusion is that random errors in rawinsonde data are not the major factor determining structure function profiles for either single observation times or the composite 24 h period. Instead, major features of these profiles should be attributed to true atmospheric variability.

5. Summary and conclusions

Structure and correlation functions have been used to statistically describe atmospheric conditions during the first regional-scale day of AVE-SESAME 1979 (10–11 April). This 24 h period coincided with the Red River Valley tornado outbreak. Rawinsonde data at 3 h intervals and separations of 250 km were used in the study. The parameters investigated were temperature, geopotential height, horizontal wind speed, and mixing ratio. Calculations were performed in both the lower and upper troposphere.

Results show that mesoscale features were important components of the atmosphere's overall structure. Small scale variability was especially prominent in patterns of mixing ratio and horizontal wind speed. Although anisotropic conditions were observed with all parameters, the isotropic assumption was best satisfied at small station separations. Analyses at individual observation times indicated that mesoscale variance was greatest after the onset of the convective outbreak. The developments of a low-level jet stream, areas of convergence in the lower troposphere, and a region of strong low-level height falls were clearly depicted in the structure function analyses. Sensitivity experiments were used to document the effects of missing data and data errors on the functional calculations. Results demonstrated that data limitations could not have produced the observed variations in structure values.

By extrapolating structure functions to zero separation distance, we made random error estimates for the rawinsonde data. These values showed good agreement with those derived using different techniques. Finally, correlation functions were used to compare the variance contained in the four parameters. Results suggested that the NWS network alone would have failed to resolve important structural features of this convectively active period.

The study demonstrates that statistical structure and correlation functions are powerful tools for monitoring atmospheric variability. They permit one to quantify information concerning meteorological patterns that are seen qualitatively on horizontal depictions. The prominence of mesoscale features during this severe storm outbreak emphasizes the need for upper-air mesoscale data. The recently launched series of geostationary satellites equipped with the VISSR Atmospheric Sounder (VAS) may be an important source of this needed information (Smith et al., 1981).
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